Manual for the Tiling Database

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1 Introduction

This site consists of a data base and search facilities to locate items in the collection. Unlike most data bases, the items are images, but ones that have

substantial structure. This structure is recorded in the data base making searching and retrieval possible.

This manual describes how the system works from the point of view of the user. It does not give implementation details.

2 Scope

The system only covers geometric patterns. Moreover, the pattern must consist of tiles which are polygons. Hence the edges are straight lines.

The other major restriction is that the tiles must be fitted together by means of local connection rules, that is, every edge has a matching edge. This does allow tilings which have a centre and hence are not repeat patterns. It does not allow substitution patterns like to one due to Penrose [2]. A small patch of a Penrose tile is possible [1], but examples are not currently included in the collection.

The initial emphasis in this collection is on classical Islamic patterns, since it seems possible to record all of these.

3 Sources

Currently, almost all of the material recorded here has been extracted from publications. Hence for every item, there is a list of publications that refers to it. The complete list of publications is in the list. Each publication is referenced by text string enclosed in square brackets.

In those cases in which the author has been provided with information that has not been published or placed on the Internet, the 'publication' [pc] is used (standing for personal communication).

Those publications that have a Pressmark are ones which have been located via the National Art Library catalogue. This catalogue can produce details in Unicode (UTF-8), which is used here. Another useful library with an on-line catalogue is that of the School of Oriental and African Studies.

If you want an ordinary publication containing the great majority of the information here, you can purchase on-line the CD/booklet: [3].

Given an actual source, transcribing this into a high-quality digital image is not straightforward. Many older artifacts are in poor condition, or sometimes the actual photograph does not have the resolution needed to see the detail. The great majority of the source material is not in colour so that the colours used here are often an invention. (Some of the colours have been chosen at random and are very poor — these will be redone in time.)

It was thought at first that the system should record artifacts as well as publications. This proved too difficult and not as worthwhile as recording publications. Publications have a recognised method of reference and have agreed dates, while artifacts do not. Hence for something like the Lindisfarne Gospel, a modern book is used rather than the actual manuscript. In this case, the British

Library has an excellent Internet viewing facility, but this is not referenced since URLs are hard to keep up-to-date. Traditional search engines should be used to locate such material.

Over the next few years, an increasing number of older publications will be available on the Internet. Many academic papers are available that way, at least from the author. However, Internet references (URLs) often vary and hence this system does not store the URL. Instead, those publications available on the Internet are marked as such. As far as possible, a check is made that the current URL can be found using a search engine.

4 Producing the digital images

Given a source of a tiling, the production of a digital image is not straightforward. Photographs or the original object may have imperfections which are not copied. Many tilings have no colour information and hence the colour has been added to aid the presentation.

The great majority of the tilings here are repeat patterns. However, the original are obviously finite and in some cases, the repetition used is unclear.

The more complex Islamic star patterns could be represented in slightly different ways and yet still be the same 'tiling'. One needs to try to recover the original concept which has typically been lost. Tony Lee has provided valuable insight into these issues and his advice has been followed here. Roughly speaking, the aim is to increase the symmetry, either of the tiling as a whole, or of individual tiles. It is also sometimes possible to re-use an existing tile in a pattern rather than introducing another similar tile.

In Book VIII of Plato's 'Republic', Socrates illustrates the concept of reality and what we perceive by an illustration of men who are chained inside a cave with a view exclusively of the inner wall, on which they see shadows of objects paraded behind them, obstructing the daylight. "Such men would not perceive truth to be anything other than the shadows of artificial objects." Here, we are trying to reconstruct the original idea using modern technology. Hopefully, more than a shadow...

5 Structural image data

The transcription process produces the graphic detail from which a drawing program can produce the images seen in this collection. In addition, structural detail is obtained, partly by computing data from the graphical representation and partly from additional information stored with the image.

One structural item is the largest angle for which all internal angles of every polygon is a multiple. This angle is computed from the image data and hence should not be in doubt. Some patterns have polygonal angles which are effectively arbitrary — and hence the computation does not provide this measure. The 'angle' is then recorded as zero.

Another structural item is very different. This is the symmetry group. The information stored is either that no symmetry exists, or one of the 17 planar symmetry groups (*× (*cm*), 2*22 (*cmm*), O (*p1*), 2222 (*p2*), 333 (*p3*), 3*3 (*p3lm*), *333 (*p3ml*), 442 (*p4*), 4*2 (*p4g*), *442 (*p4m*), 632 (*p6*), *632 (*p6m*), ×× (*pg*), 22× (*pgg*), ** (*pm*), 22* (*pmg*), *2222 (*pmm*)). Note that we are using the Conway-Thurston notation, but retaining the conventional notation in brackets (for an explanation, see PDF). For technical reasons, the image processing software is not clever enough to compute the group. Hence the value is added by hand. This implies that the possibility of an error in assigning the symmetry group cannot be ruled out. Some simple computer checks can be made, but errors could still occur. (If the symmetry group is recorded as *442 (*p4m*), but the angle above is recorded as 30 degrees, something is clearly wrong!)

Some items are computed, but cannot always be determined exactly. For instance, with a repeating pattern, the computation of whether it can be coloured with just two colours is straightforward (there are an even number of lines at each vertex). Unfortunately, for centered patterns in which the fitting rules are not one-to-one between edges, the computation is not straightforward and is not undertaken.

The conclusion from the above is that the structural image data is not perfect, but with care, is quite sufficient to locate almost all patterns in the collection.

6 Identification of a tiling

When the system is built, each tiling is given a five digit number. This is preceded by a 'T' in the output. It is possible that this number could change when the system is rebuilt. The full results for any tiling also gives a reference to the original data in the form `data`*`n/name`*. It is probably best to use this format to refer to a pattern for future use.

Unlike conventional books, this system is subject to change. Errors will be corrected, and as further information becomes available, the colouring of many tilings could be changed. Hence referring to a pattern as, say, Bourgoin Plate 118 may well be best! Note that an individual tiling may occur in several publications, but this system typically has a title for the tiling which refers to just one publication.

7 The Tree search facility

This is probably the easiest tool to use to locate a tiling. It works like a Flora - questions are asked about the tiling which leads to another question. Eventually, the number of tiling which satisfies the answers to the questions is reduced to a small enough number of display all the tilings. From this display, a final choice can be made (or you have showed that pattern being sought is not in the

collection).

The questions are fixed which means that if a mistake is made, the browser *back* command allows one to retrace ones steps and make an alternative choice.

8 The main search facility

This search uses an HTML form to specify a number of conditions that a tiling pattern must satisfy. Unlike many Internet searches, this one requires an exact match. If you request a pentagon, then the pattern has to have an exact regular pentagon — one that looks as if it might be regular, but is not, will not do.

To satisfy the conditions, all of them must be satisfied. Hence, when combining unusual conditions, it way well be that no tiling pattern satisfies the conditions you have given.

One cannot specify a search for the absence of something. Hence one may get a tiling pattern with a regular octagon even though that was not requested.

When you are uncertain about something, leave the box blank. You can leave all the boxes blank and obtain a list of the entire collection! As the collection grows, it will be necessary to select the conditions carefully so that the visual inspection of the patterns can be reduced.

Note that two search methods are provide which do not rely upon the geometry of the tiling — the text search and the search by publication.

8.1 Individual search constraints

Precise details are given here for each search constraint.

Symmetry group of tiling pattern. Do not choose a symmetry group unless you are sure which one it is! There are several publications describing the groups and their properties. The colouring and the style of the lines are ignored when determining the symmetry group.

Examples: *× (*cm*), 2*22 (*cmm*), O (*p1*), 2222 (*p2*), 333 (*p3*), 3*3 (*p3lm*), *333 (*p3ml*), 442 (*p4*), 4*2 (*p4g*), *442 (*p4m*), 632 (*p6*), *632 (*p6m*), ×× (*pg*), 22× (*pgg*), ** (*pm*), 22* (*pmg*), *2222 (*pmm*).

Can the pattern be coloured with just two colours? The system records the property as true, false, or don't know.

Examples: true, false.

If the tiling to be searched for only requires two colours, then two additional properties will be requested:

- Finite interlaces: This implies that tracing round the edges always comes back to the starting position. The number of distinct patterns for the tracing is entered here.
- Infinite interlaces: Here the tracing is never-ending. The number of such interlaces is requested.

Example: Five finite interlaces, but no infinite interlace url.

Number of sides of regular polygons This are exact regular polygons.

Examples: pentagons, enneagon.

The number of points to the first regular star polygon is The number of points must be entered, and additionally, the vertex angle can be specified. If the vertex angle is not specified, then any star polygon with that number of points will suffice. Note that strictly, we are using star-shaped polygons rather than polygons given by joining vertices of a regular polygon, see section 2.5 of [5].

Examples: Two stars, two more stars. Note that in the first example, both stars are conventional star-polygons, being [5/2] and [10/4]. The second example, both the stars are not conventional, but are star-shaped polygons.

The number of points to the second regular star polygon is Again, the number of points must be entered, and additionally, the vertex angle can be specified. Although some tilings have three or more star polygons, only two can be used in a search.

The tiling angles For the majority of tilings, all the internal angles of the polygon are multiples of a small angle. If there is no such angle, then one can enter 0 in this field.

Examples: 22.5 degrees. 15 degrees.

The two polygon condition A simple true/false property. See the reference for the details.

Examples: satisfied, similar tiling: not satisfied.

The number of polygonal shapes The system handles regular polygon and regular star polygons separately, so here, all the other polygonal tiles are considered. There are three numbers which can be entered as follows:

1. The number of tiles which have at least one mirror line.
2. The number of pairs of tiles which are mirror images of the other member of the pair.
3. The number of tiles which have no mirror image tile.

Example: 6,0,0, which is one of 33 tilings with six reflective tiles, but no other tiles.

Edge-to-edge property This property is false if a straight line edge of a polygon is joined to more than one other polygon. This property is true or false.

Examples: true, false.

Text to be searched for Associated with each tiling is a title, short description and references. A textual search is made of this information, ignoring the case.

Example: Search for the Alhambra gives several tilings including this one.

Publication search A choice can be made here for one publication.

Example: Bourgoins collection is well known (over 100 are included here) and includes: Plate 81.

Presentation options The default presentation mode is to give a small image for each tiling that matches the request. Alternatively, the title can be given instead. Lastly, and option can be given which provides the references as well (but no images).

The search options can be combined by requiring all the options to be satisfied. For instance, one can locate the 28 tilings which are in the Alhambra and have symmetry group $p4m$, of which this is one.

9 The simple search facility

This facility merely requires that the regular polygons and regular star polygons are identified by ticking the appropriate boxes.

This facility is rather different from the main search, since if the octagon box is not ticked (say), then no octagon will appear in the resulting tilings. Hence one needs to be confident of the regular (star) polygons present in the tiling.

Being very simple, a search may produce many tilings which then have to be inspected by hand to locate the one required. It is important to note that the presence of a diamond (a two-pointed star!) may not be obvious, but must be included in the search options to locate the tiling.

If you are searching for a tiling that has no regular polygon or star polygon as a tile, then this simple search with no boxes filled, will list all tilings with no such polygons.

Note that not more than five boxes should be ticked (in fact, only the first five boxes are taken — the result will yield no tilings anyway).

10 The tree search facility

This system works like a Flora in asking questions until the number of tilings satisfying the properties is small enough to display them all.

If you are not familiar with the symmetry groups, this search method should be suitable; however, it is slower to use than the main search facility.

At every stage examples are given of patterns which satisfy each of the conditions associated with the new branch in the search tree.

Start a tree search by clicking here.

11 Presentation of the search results

The default means of presenting the result is as a sequence of small GIFs which are pointers to the full details for each pattern. This is the only option provided with the simple search.

Two alternative means of presenting the results of a search are provided: Listing the text of title and position of the tilings, or giving that and additionally the references to publications. The textual presentations do not give links to the main data.

12 Tilings that are not included

If an individual tile is not a topological disc (for instance, it has an internal hole), then the software used here cannot handle it and therefore the pattern is excluded from consideration. (Currently, excluded tilings are not recorded, but that information could be added at a later date.)

Some tilings include curves. The software does not handle curves, but a sequence of lines can be used instead. Patterns which are mainly curves are not considered.

A more technical restriction applies to the individual tiling which must have local connection rules [4]. This implies that Penrose tilings cannot be handled, although quite large patches of such a tiling is possible [1]

13 Other issues

Acknowledgements and copyright are handled in the on-line HTML documentation.

References

[1] B. A. Wichmann and J Rigby. A Penrose-type Islamic Interlacing Pattern, Visual Mathematics, Volume 9, No 2, 2007. URL

[2] R Penrose: Pentaplexity. *Mathematical Intelligencer*. 1979. pp32-37.

[3] B. A. Wichmann, The World of Patterns, CD and booklet. World Scientific. 2001. ISBN 981-02-4619-6 URL (This publication has over 4,000 patterns but it a different format from the on-line system.)

[4] Steven Dworkin, Jiunn-I Shieh, Deceptions in Quasicrystal Growth, *Commun. Math. Phys.* 168, 337 - 352 (1995)

[5] B. Grünbaum and G. C. Shephard, *Tilings and Patterns*, W. H. Freeman & Co., New York, NY, 1987.

A Revision record

1. Started, 1st June 2007.
2. Revised to reflect comments from Ron Knott, 7th July 2007.
3. Revised to reflect some small improvements, 11th August 2007.
4. Hypertext link corrected, and quote from Plato added, November 4th 2007.
5. Added material on each search option, 22nd November 2007.
6. Added links to examples, 15th December 2007.
7. Minor change, 25th February 2008.
8. Changed for Version 4, 29th February 2008.
9. Changed for Version 7, 23rd June 2008.